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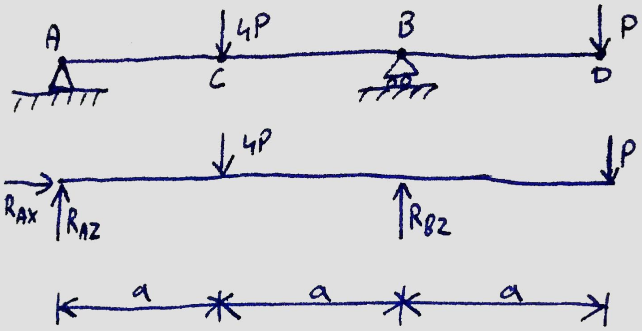
# 1A




Dane:

$P = 10,2\ kN$

$a = 1,2\ m$

Szukane:

$T(x) = ?$

$M_g(x) = ?$

$\sum F_x = 0: \quad R_{Ax} = 0$

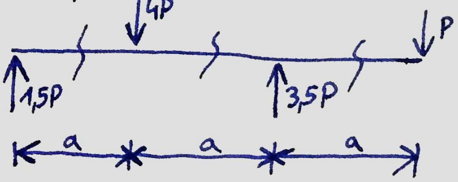
$\sum M_A = 0: \quad -4P\cdot a + R_{Bz}\cdot 2a - P\cdot 3a = 0 \Rightarrow$

$\Rightarrow R_{Bz} = \frac{7Pa}{2a} = 3,5P$

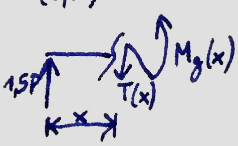
$\sum F_z = 0: \quad R_{Az} - 4P + R_{Bz} - P = 0 \Rightarrow$

$\Rightarrow R_{Az} = 5P - R_{Bz} = 1,5P$

Myślowe przecięcia:




I) $x \in (0, a)$




$\sum F_z = 0:$

$1,5P - T(x) = 0$

$T(x) = 1,5P = 15,3\ kN$

$\sum M_{pp} = 0:$

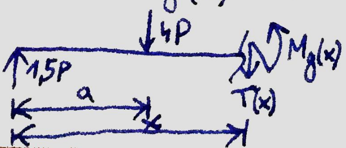
$-1,5P\cdot x + M_g(x) = 0$

$M_g(x) = 1,5P\cdot x = 15,3x\ [kNm]$

$M_g(0) = 0\ kNm$

$M_g(a) = 18,36\ kNm$

II) $x \in (a, 2a)$




$\sum F_z = 0: \quad 1,5P - 4P - T(x) = 0$

$T(x) = -2,5P = -25,5\ kN$

$\sum M_{pp} = 0: \quad -1,5P\cdot x + 4P(x-a) + M_g(x) = 0$

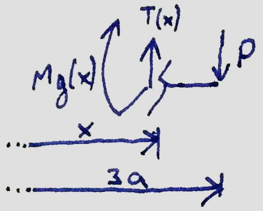
$M_g(x) = 4Pa - 2,5Px =$

$= 48,96 - 25,5\cdot x\ [kNm]$

$M_g(a) = 18,36\ kNm$

$M_g(2a) = -12,24\ kNm$

III) $x \in (2a, 3a)$




$\sum F_z = 0:$

$T(x) - P = 0$

$T(x) = P = 10,2\ kN$

$\sum M_{pp} = 0:$

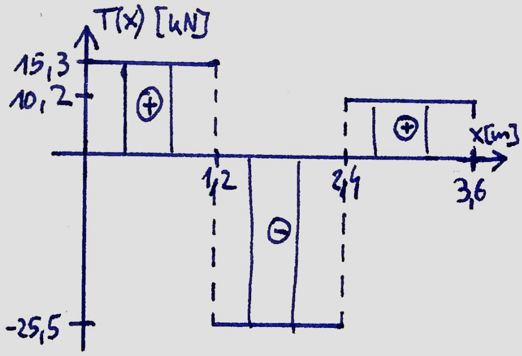
$-M_g(x) - P(3a - x) = 0$

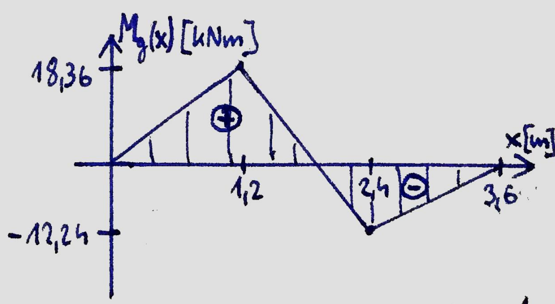
$M_g(x) = Px - 3Pa =$

$= 10,2x - 36,72\ [kNm]$

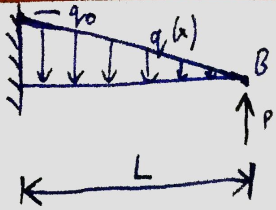
$M_g(2a) = -12,24\ kNm$

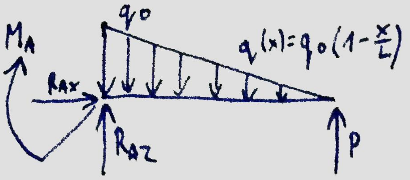
$M_g(3a) = 0\ kNm$



## 1B




**Dane:**

$q_0 = 5{,}2\ \frac{kN}{m}$

$P = 4{,}12\ kN$

$L = 2m$

**Szukane:**

$M_g(x) = ?$

$T(x) = ?$

$\Sigma F_x = 0:\ R_{Ax} = 0$

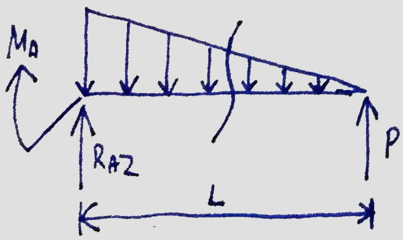
$\Sigma F_z = 0:\ R_{Az} - \underbrace{\frac{1}{2}\cdot q_0 \cdot L}_{\text{graficznie, ciężar}} + P = 0$

$R_{Az} = \frac{q_0 \cdot L}{2} - P = 1{,}08\,kN$

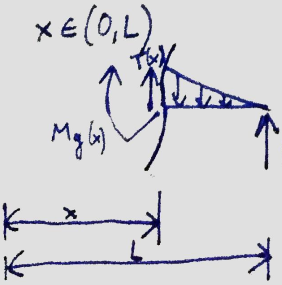
$\Sigma M_{gA} = 0:\ -M_A - \frac{1}{2}\cdot q_0 \cdot L \cdot \frac{L}{3} + P\cdot L = 0$

$M_A = PL - \frac{q_0 L^2}{6} \approx 4{,}77\,kNm$

**Myślowe przecięcie:**




**I)** $x \in (0, L)$




$\Sigma F_z = 0:$

$T(x) - \underbrace{\frac{1}{2} q_0 \left(1 - \frac{x}{L}\right)(L - x)}_{\text{graficznie, ciężar}} + P = 0$

$T(x) = \frac{q_0 (L-x)^2}{2L} - P =$

$= 1{,}3(2-x)^2 - 4{,}12\ [kN]$

$T(0) = 1{,}08\,kN \quad T(\frac{L}{2}) = -2{,}82\,kN$

$T(L) = -4{,}12\,kN$

$\Sigma M_{pp} = 0:$

$-M_g(x) - \frac{1}{2} q \left(1 - \frac{x}{L}\right)\cdot(L-x)\cdot\frac{(L-x)}{3} +$

$+ P(L-x) = 0$

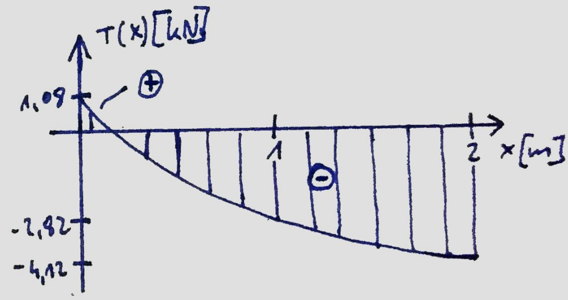
$M_g(x) = P(L-x) - \frac{q_0 (L-x)^3}{6L} =$

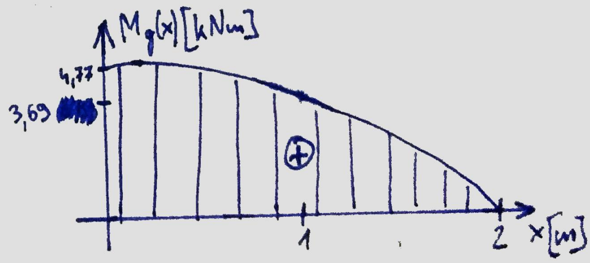
$= 4{,}12(2-x) - \frac{5{,}2\,(2-x)^3}{12}\ [kNm]$

$M_g(0) = 4{,}77\,kNm$

$M_g(L) = 0\,kNm$

$M_g(\frac{L}{2}) = 3{,}69\,kNm$



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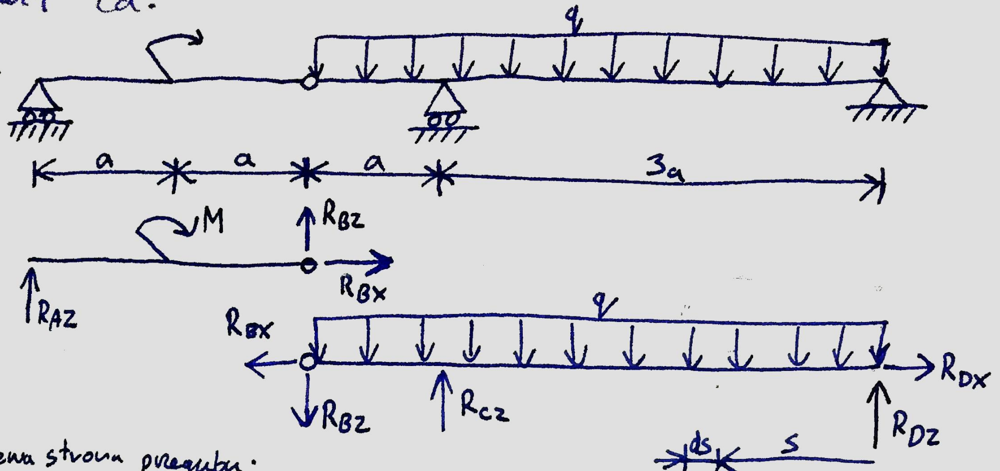
1C




Dane:

$q = 10{,}12\ \frac{kN}{m}$

$M = 2{,}2\ kNm$

$a = 0{,}5\ m$

Lewa strona przegubu:

$\Sigma F_x = 0: \quad R_{BX} = 0$

$\Sigma M_A = 0: \quad -M + R_{BZ} \cdot 2a = 0$

$$R_{BZ} = \frac{M}{2a} = \frac{2{,}2}{2 \cdot 0{,}5} = 2{,}2\,kN$$

$\Sigma F_z = 0: \quad R_{AZ} + R_{BZ} = 0$

$$R_{AZ} = -R_{BZ} = -2{,}2\,kN$$

Prawa strona przegubu:

$\Sigma F_x = 0: \quad -R_{BX} + R_{DX} = 0$

$$R_{DX} = 0$$

$\Sigma M_D = 0: \quad \int_0^{4a} s\, q\, ds - R_{CZ} \cdot 3a + R_{BZ} \cdot 4a = 0$

$$R_{CZ} = \frac{\left[\frac{q s^2}{2}\right]_0^{4a} + R_{BZ} \cdot 4a}{3a} = \frac{\frac{q \cdot 16a^2}{2} + R_{BZ} \cdot 4a}{3a} =$$

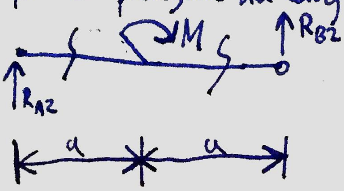
$$= \frac{8qa}{3} + \frac{4R_{BZ}}{3} = 16{,}43\,kN$$

$\Sigma F_z = 0: \quad -R_{BZ} + R_{CZ} - \int_0^{4a} q\, ds + R_{DZ} = 0$

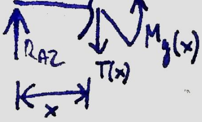
$$R_{DZ} = R_{BZ} - R_{CZ} + q \cdot 4a = \frac{4qa}{3} - \frac{R_{BZ}}{3} =$$

$$= 6{,}01\,kN$$

Myślowe przecięcia dla lewej strony przegubu:




I) $x \in (0, a)$




$\Sigma F_z = 0: \quad R_{AZ} - T(x) = 0$

$$T(x) = R_{AZ} = -2{,}2\,kN$$

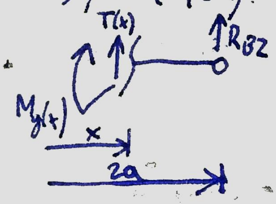
$\Sigma M_{pp} = 0: \quad -R_{AZ} \cdot x + M_g(x) = 0$

$$M_g(x) = R_{AZ} \cdot x = -2{,}2x\ [kNm]$$

$$M_g(0) = 0\ kNm$$

$$M_g(a) = -1{,}1\ kNm$$

II) $x \in (a; 2a)$




$\Sigma F_z = 0: \quad T(x) + R_{BZ} = 0$

$$T(x) = -R_{BZ} = -2{,}2\,kN$$

$\Sigma M_{pp} = 0: \quad -M_g(x) + R_{BZ}(2a - x) = 0$

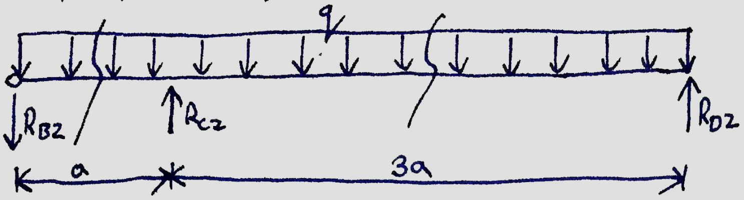
$$M_g(x) = R_{BZ}(2a - x) =$$

$$= 2{,}2 - 2{,}2x\ [kNm]$$

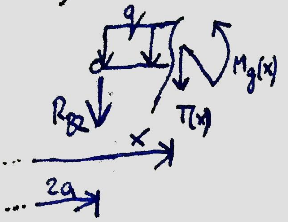
$$M_g(a) = 1{,}1\ kNm$$

$$M_g(2a) = 0\ kNm$$

Myślowe przecięcia prawej strony przegubu:




III) $x \in (2a, 3a)$




$$\sum F_z = 0: \quad -R_{BZ} - \int_{2a}^{x} q\,dx - T(x) = 0$$

$$T(x) = -R_{BZ} - q(x-2a) =$$
$$= -R_{BZ} + q\,2a - qx =$$
$$= 7{,}92 - 10{,}12 \cdot x \,[kN]$$

$T(3a) = 9{,}17\,kN$

$T(6a) = -6{,}01\,kN$

$M_g(3a) = -2{,}36\,kNm$

$M_g(6a) = 0\,kNm$

$M_g(4{,}5a) = 1{,}67\,kNm$

$$\sum M_{pp} = 0: \quad +R_{BZ} \cdot (x-2a) + \int_{2a}^{x} (x-2a)\,q \cdot dx + M_g(x) = 0$$

$$M_g(x) = -R_{BZ}(x-2a) - \left[q\left(\frac{x^2}{2} - 2ax\right)\right]_{2a}^{x} =$$
$$= -R_{BZ} \cdot (x-2a) - q\left(\frac{x^2}{2} - 2ax\right) + q\left(\frac{4a^2}{2} - 4a^2\right) =$$
$$= 2R_{BZ}a - 2qa^2 + (-R_{BZ} + q \cdot 2a) \cdot x - \frac{q}{2} \cdot x^2 =$$
$$= -2{,}86 + 7{,}92x - 5{,}06x^2 \,[kNm]$$

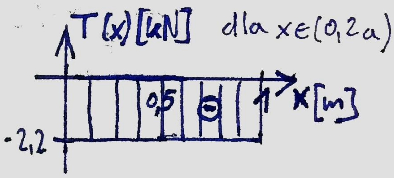
$T(2a) = -2{,}2\,kN$

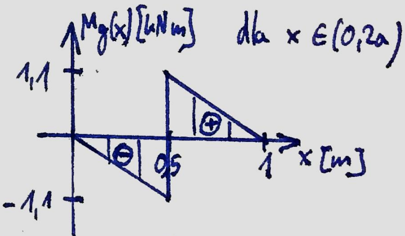
$T(3a) = -7{,}26\,kN$

$M_g(2a) = 0\,kNm$

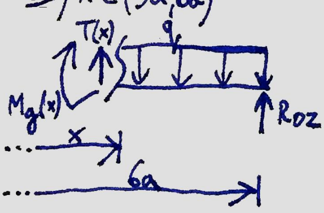
$M_g(2{,}5a) = -0{,}87\,kNm$

$M_g(3a) = -2{,}36\,kNm$



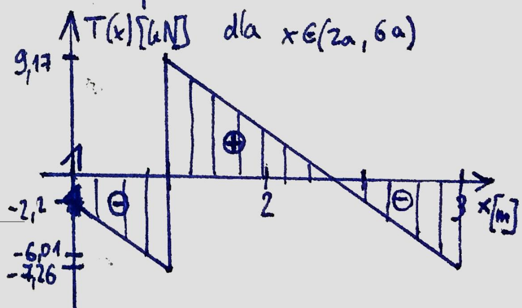




IV) $x \in (3a, 6a)$



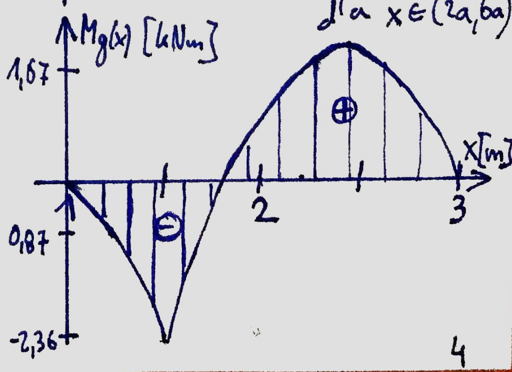

$$\sum F_z = 0: \quad T(x) - \int_{x}^{6a} q\,dx + R_{DZ} = 0$$

$$T(x) = q(6a - x) - R_{DZ} =$$
$$= 24{,}35 - 10{,}12x \,[kN]$$




$$\sum M_{pp} = 0: \quad -M_g(x) - \int_{x}^{6a} (6a - x)\,q\,dx + R_{DZ} \cdot (6a - x) = 0$$

$$M_g(x) = R_{DZ}(6a - x) - \left[q\left(6ax - \frac{x^2}{2}\right)\right]_{x}^{6a} =$$
$$= R_{DZ}(6a - x) - q\left(36a^2 - \frac{36a^2}{2}\right) + q\left(6ax - \frac{x^2}{2}\right) =$$
$$= R_{DZ} \cdot 6a - 18qa^2 + (q \cdot 6a - R_{DZ}) \cdot x - \frac{q}{2} \cdot x^2 =$$
$$= -27{,}5 + 24{,}35x - 5{,}06x^2 \,[kNm]$$



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2. z hipotezy Hubera - Misesa - Hencky'ego:

$$\sigma_{red} = \sqrt{\frac{1}{2}\left((\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2\right)} \xrightarrow[\sigma_z=0]{\sigma_y=0} |\sigma_x|$$

zatem warunek bezpieczeństwa jest równoważny:

$$k_r \geq \sigma_{red} \iff k_r \geq |\sigma_x|$$

$$\sigma_x(z) = -\frac{M_y}{I_y}\cdot z$$ ← maksymalne wartości dla maksymalnych $M_y$ oraz $z$

$$\sigma_{x\,max} = -\frac{M_{g\,max}}{I_y}\cdot z_{max} = -\frac{M_{g\,max}}{I_y}\cdot\frac{h}{2}$$

$$\sigma_{x\,min} = -\frac{M_{g\,max}}{I_y}\cdot z_{min} = -\frac{M_{g\,max}}{I_y}\cdot\left(-\frac{h}{2}\right) = \frac{M_{g\,max}}{I_y}\cdot\frac{h}{2}$$

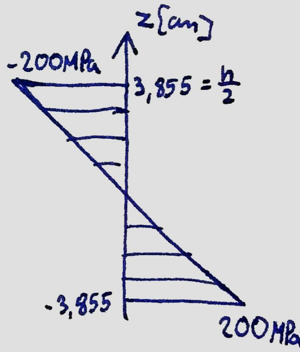
Stąd maksymalna wartość $|\sigma_x|$ (równa $k_r$) to:

$$\left(|\sigma_x|\right)_{max} = \frac{M_{g\,max}}{I_y}\cdot\frac{h}{2}$$

$$I_y = \frac{a h^3}{12} = \frac{1{,}2\cdot h^4}{12} = 0{,}1\cdot h^4$$

$$k_r = \left(|\sigma_x|\right)_{max} \iff k_r = \frac{M_{g\,max}}{I_y}\cdot\frac{h}{2} \iff k_r = \frac{M_{g\,max}}{0{,}1\cdot h^4}\cdot\frac{h}{2} = \frac{M_{g\,max}}{0{,}2\cdot h^3}$$

$$h = \sqrt[3]{\frac{M_{g\,max}}{0{,}2\cdot k_r}} = \sqrt[3]{\frac{18{,}36\cdot 10^3\,Nm}{0{,}2\cdot 200\cdot 10^6\,Pa}} = 7{,}71\ cm$$



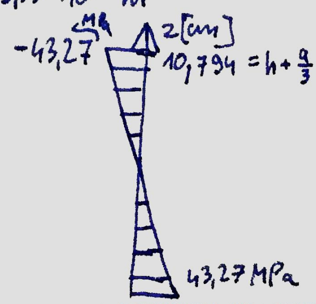

$$I_{y2} = \frac{h\cdot\left(2h+\frac{2a}{3}\right)^3-\left(h-\frac{a}{6}\right)(2h)^3}{12} = \frac{h(2h+0{,}8h)^3-(h-0{,}2h)(2h)^3}{12} = \frac{21{,}952h^4-6{,}4h^4}{12} = 1{,}296h^4 =$$

$$= 4579{,}55\ cm^4$$

$$\sigma_{x\,max2} = -\frac{M_{g\,max}}{I_{y2}}\cdot z_{max2} = -\frac{M_{g\,max}}{I_{y2}}\left(h+\frac{a}{3}\right) = -\frac{M_{g\,max}\cdot 1{,}4h}{I_{y2}} = -\frac{18{,}36\cdot 10^3\,N\,m\cdot 1{,}4\cdot 0{,}0771\,m}{4579{,}55\cdot 10^{-8}\,m^4} = -43{,}27\,MPa$$

$$\sigma_{x\,min2} = -\frac{M_{g\,max}}{I_{y2}}\cdot z_{min2} = -\frac{M_{g\,max}}{I_{y2}}\left(-\left(h+\frac{a}{3}\right)\right) = \frac{M_{g\,max}\cdot 1{,}4h}{I_{y2}} = \frac{18{,}36\cdot 10^3\,Nm\cdot 1{,}4\cdot 0{,}0771\,m}{4579{,}55\cdot 10^{-8}\,m^4} = 43{,}27\,MPa$$

Naprężenia w przekroju prostokątnym są wyraźnie większe niż w ceowniku:




$$\begin{cases}\sigma_{x\,max} < \sigma_{x\,max2}\\ \sigma_{x\,min} > \sigma_{x\,min2}\end{cases} \iff \begin{cases}-200\,MPa < -43{,}27\,MPa\\ 200\,MPa > 43{,}27\,MPa\end{cases}$$